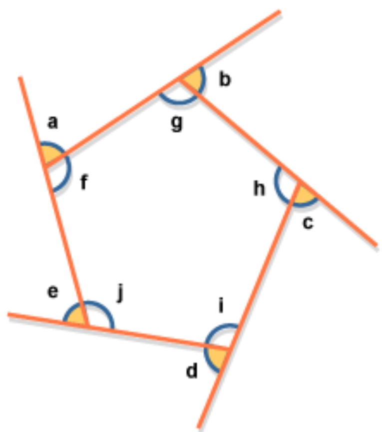
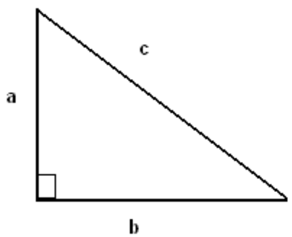
Quantitative Review
Geometry - Polygons

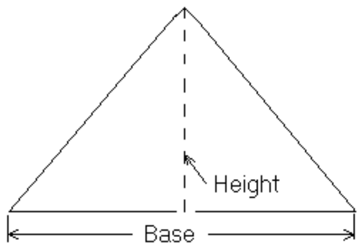
- **Polygons and Interior Angles:**
  - The sum of the interior angles of a polygon depends on the number of sides (n) the polygon has:
  - **(n – 2) x 180 = Sum of Interior angles of a polygon**



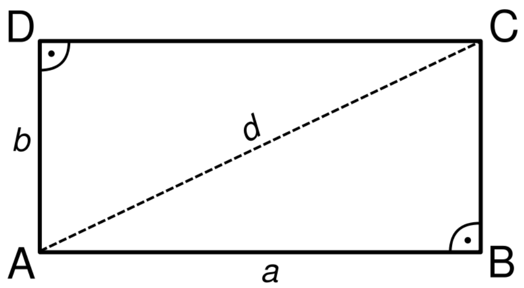
  - In the figure above, (f + g + h + i + j) = 540°, because the polygon has 5 sides
  - Every polygon can be cut in x triangles, and the sum of each triangle's interior angles is 180°. This is an alternative method to find the sum of a polygon's interior angles

- **Polygons and Area**
  - 1) Area of a TRIANGLE = **(Base x Height) / 2**

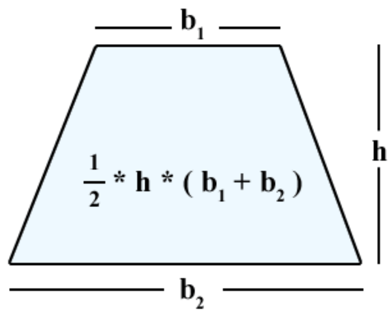


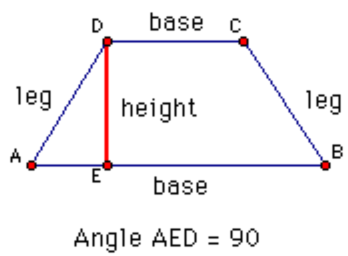


  - 2) Area of a RECTANGLE: **Length x Width**



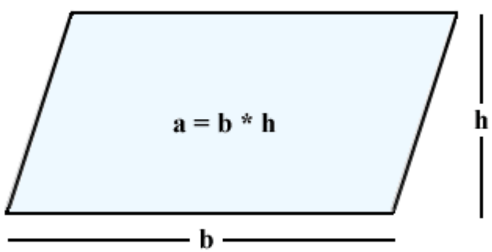
- **Polygons and Area**
  - 3) Area of a TRAPEZOID = **((Base1 + Base 2) x Height) / 2**



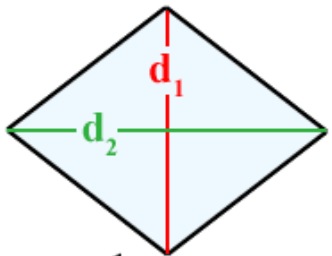


    - Notice that the height is also a line perpendicular to the two bases, inside the trapezoid
    - **Sometimes, you will have to draw a right triangle. If you know both base 1 and 2, you know the base leg. If you know the outer leg, you have 2 dimensions and can find the third, which is the height, via pythagorean theorem!!!**

  - 4) Area of a PARALLELOGRAM: **Base x Height**

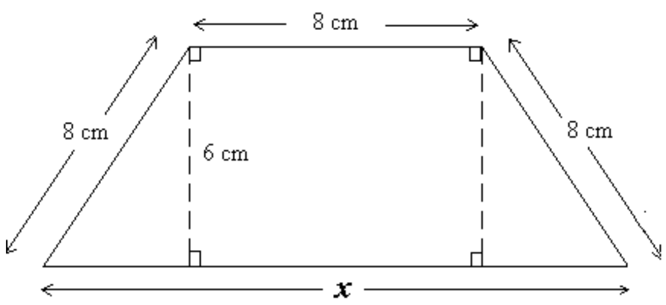


- **Polygons and Area**
  - 5) Area of a RHOMBUS = **(Diagonal 1 x Diagonal 2) / 2**

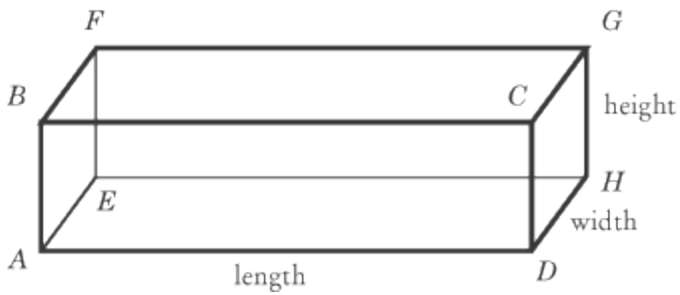


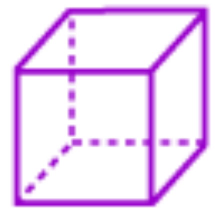
$$a = \frac{1}{2} * d_1 * d_2$$

  - GMAT may require you to divide some shapes. Notice, for example, that a trapezoid can be cut into 2 right triangles and 1 rectangle:



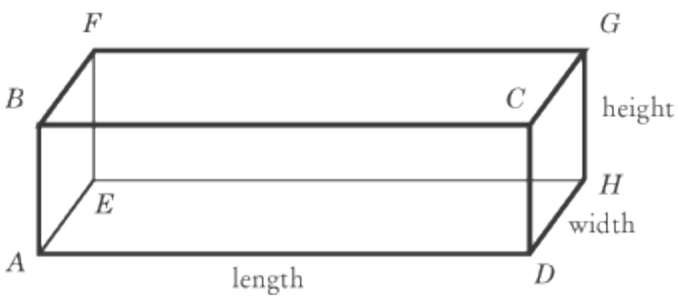
- **3 Dimensions: Surface Area**

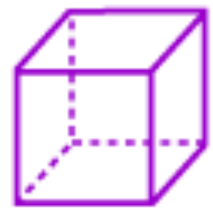




  - **Surface Area = the SUM of the areas of all of the faces**
  - **Rectangular Solid: 2(Base x Height) + 2 (Width x Height) + 2 (Base x Width)**
  - **Cube: 6 x $(\text{Side})^2$**

- **3 Dimensions: Volume**

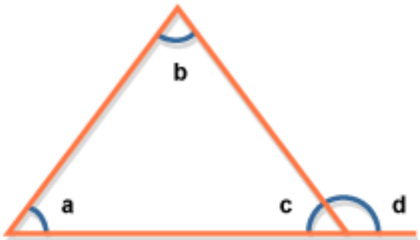




  - **Volume = Length x Width x Height**
  - **Volume of a Cube: Side^3**

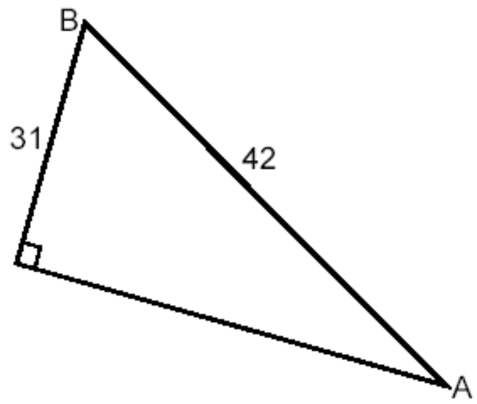
  - **GMAT TRICK: How many Books, each with a volume of 100 in^3, can be packed into a crate with a volume of 5,000 in^3?**
    - **You cannot answer to this question without knowing the exact dimensions of each book.**
    - **Remember: if you are fitting 3 dimensional objects into other 3-dimensional objects, knowing the respective volumes is not enough**

- **The Angles of a Triangle**
  - **(1) The sum of the three angles of a triangle equals 180**
  - **(2) Angles correspond to their opposite sides**



  - **If angle a = angle b, their opposite sides will be equal as well. Also, the biggest angle of a triangle will be opposite to the biggest side of this triangle.**
  - **(3) The sum of any two sides of a triangle MUST BE GREATER THAN the third side. <u>So, if you are given two sides of a triangle, the length of the third side must lie between the difference and the sum of the two given sides</u>**

- **The Angles of a Triangle**
  - **(3) cont.**



  - **The other side: 11 < x < 73 (supposing this is not a right triangle)**

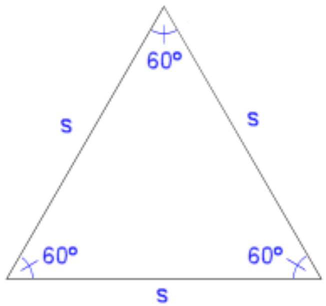
  - **Common Right Triangles**
    - **3-4-5 and its key multiples: 6-8-10, 9-12-15, 12-16-20**
    - **5-12-13 and its key multiples: 10-24-26**
    - **8-15-17**

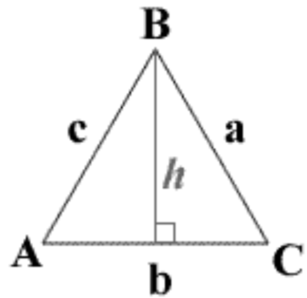
- **Isosceles Triangles and the 45-45-90 triangle**
  - **The 45-45-90 triangle is a special triangle with 2 equal sides and a relation between each side. If you are given one dimension on a 45-45-90 triangle, you can find the others.**

  - **Relationship between sides:**
  - **45° - 45° - 90°**
  - **Leg - leg - hypotenuse**
  - **1 : 1 :** $\sqrt{2}$

  - **A 45-45-90 is exactly half of a square. Two 45-45-90 triangles put together make up a square. So, if you are given the diagonal of a square, you can find the side by using the relation above**

- **Equilateral Triangles and the 30-60-90 Triangle**





- **This triangle, when cut in half, yields two equal 30-60-90 triangles. The long leg will be the height, the hypotenuse is the equilateral triangle's side and the short leg is half of the equilateral triangle's half.**

- **Relationship between sides:**
- **30° - 60° - 90°**
- **Short leg - long leg - hypotenuse**
- **1 : sq-rt 3 : 2**
- **So, if you have either the side or the height of a equilateral triangle, you can find its area**

Quantitative Review
Geometry – Triangles & Diagonals

- **Diagonals of other polygons**
  - **Diagonal of a square: d = s . (Sq-rt 2)**
  - **Main Diagonal of a Cube: d = s . (sq-rt 3)**
- **To find the diagonal of a rectangle, you must know either both sides or the length of one side and the proportion from this to the other side**
- **To find the diagonal of a rectangular solid, if you know the 3 dimensions, you can use pythagorean theorem twice:**

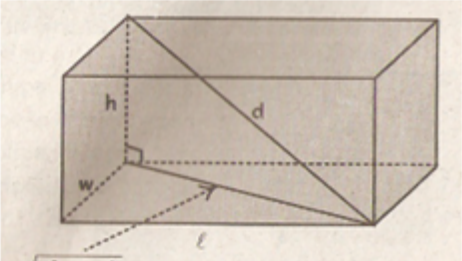


Figure 7.7

- **<u>First, use pythagorean theorem with the length and the width to find the diagonal of the bottom face. Then, use pythagorean theorem again to find the main diagonal. The sides for this second pythagorean will be: the height, the bottom face diagonal and the main diagonal.</u>**

- **Similar Triangles**
  - **Triangles are defined as similar if all their corresponding angles are equal and their corresponding sides are in proportion.**

  - **If two similar triangles have corresponding side lengths in ratio a:b, then their areas will be in ratio a^2:b^2**

- **Triangles and Area, revisited**
  - You can designate any side of a triangle as the base. So, you have 3 ways to calculate the area of a triangle, depending on which side is considered the base. Obviously, those 3 calculations will all yield the same result
  - In right triangles, if you choose one of the legs as the base, the other leg will be the height. If you choose the hypotenuse as the base, you will have to find the height.
  - **The area of an equilateral triangle of side S is equal to (S^2 . (st-rt 3)) / 4.** This is because an equilateral triangle can be cut in 2 30-60-90 triangles, and the proportion of the height will be S . (sq-rt 3) / 2.

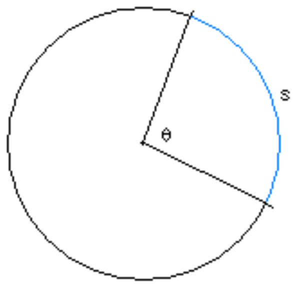
– Circles

- A circle: a set of points in a plane, equidistant from a fixed center

- Any line segment that connects the center to a point in the circle is called **radius**. Any line segment that connects two points on a circle is called **chord**. Any **chord** that passes through the center is called **diameter. Diameter = 2.radius**

- **Circumference: $C = 2\pi r$**
  - Some GMAT problems will require you to calculate π. In those, use 3 or 22/7.
  - **A full revolution of a spinning wheel means a point on the edge of a wheel travels one circumference.**
  - **If a wheel spins at 3 revolutions per second and its diameter is 10 units, a point on the edge will travel at a rate of 30π/s (Circumference = 10 π, and the point travel 3 circumferences per second).**

- **Circles: Arc Length**
  - A portion or distance ON a circle is called ARC.



  - The length of Arc AxB can be calculateded by:
    - **Length AxB = Circumference . Angle/360**
- **Perimeter of a Sector**
  - The perimeter of a sector is **ARC + 2 Radius**

- **To find either the Arc Length or the perimeter of a sector, all you need is the radius plus the angle of the sector**

- **Circles: Area of a Circle**
- **$A = \pi r^2$ ⟶ All we must know is the radius**

- **Area of a Sector**
  - **Area of a Sector = $\pi r^2 \cdot (x°/360°)$**

- **An inscribed angle is equal to half of the arch it intercepts. You can be asked this property by having the outer angle different then half the "central angle". If that is the case, the point of the "central angle"**

- **Circles: Inscribed Triangles**
- A triangle is said to be inscribed in a circle if **all of the vertices are points on the circle**
- Main property: **if one of the sides is the diameter, the triangle IS a right triangle**. Conversely, any inscribed right triangle has the diameter as one of its sides.
- A right triangle can be opposed to a semicircle. If you need to calculate that arc, it is 180°

- **Take care: A triangle inscribed in a semicircle doesn't have the same properties as a properly inscribed triangle.**

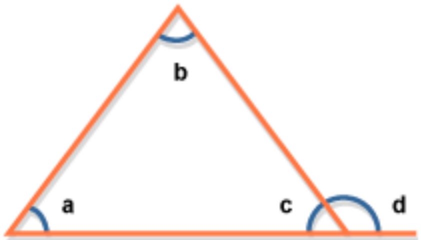
- **Cylinders**
- **Surface Area: $2\pi(r^2) + 2\pi rh$**
- **Volume: $V = \pi r^2 h$**
- **To find either the surface area or the volume, you only need the radius and the height.**

- **GMAT TRICK: Two cylinders can have the same volume but fit a different larger object. Different combinations of radius and heights can produce the same volume but very different cylinders.**

Quantitative Review
Geometry – Lines and Angles

- **Lines and Angles**
  - **Straight Line: Shortest distance between 2 points. Angle: 180°**
  - **Parallel lines never intercept**
- **Intersecting Lines: Properties**
  - **The sum of interior angles is 360**
  - **Interior angles that combine to form a line sum 180°**
  - **Angles found opposite each other where two lines intersect are equal**
- **These properties apply to more than two lines that intersect at a point**

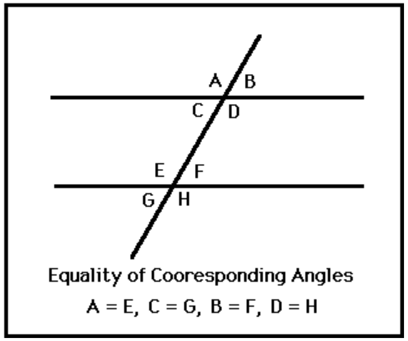
- **Exterior angles of a triangle**



  - a + b + c = 180;
  - c + d = 180
  - **d = a + b**
  - **This property is frequently tested on GMAT**

- **Parallel lines cut by a transversal:**
  - **Sometimes parallel lines cut by a transversal appear when a rectangle, parallelogram, rhombus or trapezoid is cut in half by a diagonal**

– Parallel lines cut by a transversal:



In this figure, we have only two angles, A and B. All the other acute angles equal B, and the obtuse angles equal A

- **The eight angles in the diagram above are actually 2, and a+b = 180**
- **All the acute angles (<90°) are equal.**
- **All the obtuse angles (90°<x<180°) are also equal**
- **Any acute angle is supplementary to any obtuse angle (a+b = 180)**

- GMAT can disguise the lines, by not drawing them completely. Extending them can help you to find out whether they are transversal or parallel**. GMAT uses the symbol II to indicate 2 lines are parallel. MN II OP means those segments are parallel**

Quantitative Review
Geometry – Coordinate Plan

– Coordinate Plan

- (a,b): a = x-coordinate, and b = y-coordinate

– The Slope of a Line

- The slope is defined as "rise over run": $(Y_1 - Y_2) / (X_1 - X_2)$
- In a line, any two other points have different "rise" and different "run", but the slope will always be the same.
- There are 4 types of slopes: positive slope, negative slope, zero slope and undefined slope.
- **A line with positive slope rises upward from left to right. A line with negative slope falls downward from left to right. A horizontal line has zero slope, and a vertical line has undefined slope. The x-axis has zero slope, and the y-axis has undefined slope.**

- **The intercepts of a line**
  - An intercept is a point where the line intersects a coordinate axis
  - X-Intercept = (x,0)
  - Y-Intercept = (0,y)
  - **To find x-intercepts, plug in 0 for y. To find y-intercepts, plug in 0 for x**
- **Slope-intercept Equation: <u>y = mx + b</u>**
    - **m = slope**
    - **b = y-intercept**
  - **Horizontal and vertical lines**
    - **Horizontal: y = some number**
    - **Vertical: x = some number**

- **Step by Step: from 2 points to a line**
  - **(1) Find the slope with rise over run**
  - **(2) Plug in the Slope**
  - **(3) Solve for b by plugging the coordinates of one point**
  - **(4) Write the equation in the form <u>y = mx + b</u>**
    - Example: Point 1 = (5, -2) - Point 2 = (3,4)
    - (1) Rise over run: (-2 - 4) / (5 – 3) = -3
    - (2) y = -3x + b
    - (3) 4 = -3.3 + b --- b = 13
    - (4) y = -3x + 13
  - **If GMAT gives you one point and one intercept, don't forget: an intercept is also a point, so you are able to find the equation**

Quantitative Review
Geometry – Coordinate Plan

- **Distance between two points**
  - **(1) Draw a right triangle connecting points**
  - **(2) Find the lengths of the 2 legs of the triangle**
  - **(3) Use pythagorean theorem**
    - Example: what is the distance between (1,3) and (7,-5)?
    - First leg = 6 (7-1); second leg = 8 (3- (-5)).
    - Pythagorean: $x^2 = 6^2 + 8^2$ --- x = 10

- Quadrants
  - **If you are required to find out which quadrant a given line passes two and you have the equation, set x and y to zero to find the two intercepts and draw the line. This is the fastest way**

Quantitative Review
Geometry – Coordinate Plan

- **Perpendicular Bisectors**
  - The perpendicular has the negative reciprocal slope of the line segment it bisects.
  - To find the equation of a perpendicular bisector:
  - (1) Find the slope of the line segment
  - (2) Find the slope of the perpendicular bisector (reciprocal)
  - (3) Find the midpoint of AB
  - (4) Find b for the bisector, by plugging the midpoint of AB
    - Example: Find the perpendicular bisector of line with points (2,2) and (0,-2)
    - Slope = 2 - (-2) / 2 – 0 = 2; Slope of the bisector = -1/2
    - Midpoint = (1,0)
    - Plugging: 0 = -1/2 . 1 + b ---- b = ½
    - **Equation of the perpendicular bisector: y = -1/2x + 1/2**

Quantitative Review
Geometry – Coordinate Plan

- **Parallel and perpendicular lines**
  - **Parallel lines have equal slopes. Perpendicular lines have negative reciprocal slopes!!! VERY IMPORTANT PROPERTY**
- **The intersection of 2 lines**
  - **If lines intersect, both equations at the intersect point are true. That is, the ordered pair (x,y) solves both equations**
    - At what point lines $y = 4x + 10$ and $2x + 3y = 26$ intersect?
    - To quickly solve this, replace $y = 4x + 10$ in the second equation.
    - $x = 4$, $y = 6$.
  - **If two lines in a plan do not intersect, they are parallel no pair of numbers (x,y) will satisfy both equations. Another possibility: both equations are the same. Then, infinitely many points will solve both equations.**
  - **To test whether a point lies on a line, just test it by plugging the numbers into the equation.**

- **Coordinate plan: Difficult question**
  - **What are the coordinates for the point on line AB that is 3 times as far from A as from B, and that is between points A and B, knowing that A = (-5,6) and B (-2,0)?**
  - The total x-axis distance from one point to the other is 3 (-5 - -2)
  - The total y-axis distance from one point to other is 6 (6 – 0)
  - Consider that the distance between A and B is 4x, and we need a point which is 3x distant from A and x distance from B.
  - Therefore, 3x = 4 --- x = 0.75. The point is located on x-axis -2.75
  - 4x = 6, x = 1.5. The point is located on y-axis 1.5
  - **The point is (-2.75, 1.5)**

Quantitative Review
Geometry – Coordinate Plan

- **Advanced Geometry**
  - **Maximum Area of a quadrilateral**
  - Of all quadrilaterals with a given perimeter, the square has the largest area. Conversely, of all the quadrilaterals with a given area, the square is the one with the smaller perimeter.
  - If you are given 2 sides of a triangle or a parallelogram, you can maximize the area by placing those two sides perpendicular
  - **Quadratics**
  - A quadratics function graph ($y = ax^2 + bx + c$) is always a parabola. If $a>0$, the parabola opens upward. If $a<0$, the parabola opens downward. If $|a|$ is large, the curve is narrow. If not, the curve is wide.
  - $b^2 - 4ac$ is the discriminant. If $b^2 - 4ac >0$, the function has 2 roots. If $b^2 - 4ac = 0$, the function has 1 root. If $b^2 - 4ac < 0$, the function produces no roots.

Quantitative Review
Algebra – Basic Equations

- **Mismatch problems**
  - GMAT may induce you to think one equation has no solution by giving you 3 variables and 2 equations. **You must try to solve each of these problems, specially in data sufficiency**. Sometimes, you can solve a problem for one variable but not for the others.
  - If there are any non-linear terms in an equation, there will <u>usually</u> be two or more solutions. Double check each one anyway.

- **Combo Problems**
  - If GMAT asks you for x + y instead of only x or y, never try to solve for the isolated variables. It will almost always be much easier to isolate the combo and get the answer.
  - Look for similarities in the numerator and denominator. You can cancel variables or similar numbers. This will save you a lot of time.

– Combo Problems

- Example: What is 2/y/4/x?
  - (1) (x + y)/y = 3 (2) x + y = 12
  - 2/y/4/x = 2/y . x/4 = 2x/4y = 1/2 . x/y (we have isolated x/y)
  - Working on (1): (x + y)/y = 3 = x + y = 3y, x = 2y, 2 = x/y
  - **Now it is easy to notice that, if x/y = 2, 1/2 . x/y = 1.**
  - **Equation 2 is insufficient, as its not possible to isolate x/y. Answer: A**

- Again: the key to solve some combos is to try to find similarities between 2 equations, instead of trying to solve them. Isolating terms instead of working with single variable is essential.

- **Absolute Value Equations**
  - Three steps to solve absolute value equations:
  - (1) Isolate the abs expression: $12 + |w - 4| = 30 \rightarrow |w - 4| = 18$
  - (2) Once you have a $|x| = a$ equation, you know that $|x| = +/- a$. So, remove the brackets and test both cases: $w - 4 = 18$, $w = 22$; $w - 4 = -18$, $w = -22$.
  - (3) Check in the original equations if both solutions are valid.
  - It is very important to check both values. Some data sufficiency problems may seem insufficient as you would have 2 answers. Although, when you check both, you may find that one solution is invalid, so the alternative is sufficient.

– Exponential Equations

- **Even exponents are dangerous, because they hide the sign of the base. For any x, $\sqrt{x^2} = |x|$. The equation $x^2 = 25$ is the same as $|x| = 5$.**
- $x^2 = 0$ has only one solution. $X^2 = -9$ has no solutions, as squaring cannot produce a negative number.
- If an equation has even and odd exponents, treat it as dangerous. It probably has more than 1 solution.
- **Same base or same exponent:** Try always to have the same base or the same exponent on both sides of an equation. This will allow you to eliminate the bases or exponents and have a single linear equation.
- **This rule does not apply when the base is 0, 1 or -1. The outcome of raising those bases to power is not unique ($0 = 0^3 = 0^{23}$).**

- **Eliminating roots**
  - The most effective way to eliminate roots is to square both sides. **Don't forget: whenever you do that, you need to check both solutions in the original equation**
- **Quadratic equations**
  - A quick way to work with quadratics is to factor them. If you have the equation $ax^2 + bx + c$, when a=1:
    - **Find two integers whose product equals c and whose sum equals b**
    - **Rewrite the equation in the form $(x + k)(x + w)$, where k and w are those two numbers which resulted in the product of c and in the sum of b.**
  - **Disguised Quadratics**
    - **When you find an equation similar to $3w^2 = 6w$, don't forget that dividing both sides by 3w will cause you to miss one solution! If you factor, you get $w(3w – 6) = 0$, so $w = 0$ is also a solution!**

- **Disguised Quadratics**
  - $36/b = b - 5$: This is a quadratic, as if you multiply both sides by b you get $36 = b^2 - 5b$.
  - The equation $x^3 + 2x^2 - 3x = 0$ **can be solved**. Factoring will result on $x(x^2 + 2x - 3) = 0$, so 0 is one solution and the factored quadratic will have two roots. **The equation has 3 solutions.**
  - **If you have a quadratic equation equal to 0 and you can factor an x out of the expression, x=0 is one of the solutions.**
  - **You are only allowed to divide an equation by a variable/ expression if you know that this variable/expression does not equal 0.**
- **Distributing factored equations**
  - **FOIL: F**irst terms, **O**uter terms, **I**nner terms, **L**ast terms
    - **(x + 7) (x – 3): F:** x.x **+ O:** x.-3 **+ I:** 7.x **+ L:** 7.-3
    - The result is: $x^2 - 3x + 7x - 21$ **=** $\mathbf{x^2 + 4x - 21}$

Quantitative Review
Algebra – Factoring / Distributing

- **Factoring / Distributing**
  - **If you encounter a quadratic equation, factoring may help you. If you encounter a factored equation, try distributing it. Never forget that $(x + k)(x – m) = 0$ means <u>$x = -k$</u> and <u>$x = m$</u>.**
  - **Zero in the denominator: Undefined**
  - For the equation $(x^2 + x – 12) / (x – 2) = 0$, you cannot multiply both sides for $x – 2$, and also, $x – 2$ cannot be zero. So, the solution will be the solution of the equation in the numerator: **<u>$x = -4$ and $x = 3$</u>**.
  - **Be careful when the denominator has x!!**
- **The 3 special factors**
  - **Those appear very often on GMAT. Recognize them instantly!**
  - **1) $x^2 - y^2 = (x + y)(x – y)$**
  - **2) $x^2 + 2xy + y^2 = (x + y)^2 = (x + y)(x + y)$**
  - **3) $x^2 - 2xy + y^2 = (x - y)^2 = (x - y)(x - y)$**

- **Disguised forms of common factored expressions:**
  - **$a^2 - 1 = (a + 1)(a - 1)$**
  - **$a^2 + b^2 = 9 + 2\,ab = a^2 - 2ab + b^2 = 9 = (a - b)^2 = 9 = a – b = +/- 3$**
  - **$(x^2 + 4x + 4) / (x^2 - 4) = (x + 2)(x + 2)/(x + 2)(x – 2) = (x + 2)/(x – 2)$**
  - **Attention: you can only simplify the equation above if you know that $x \neq 2$. Otherwise, the equation is undefined**

- **Formulas**
  - **Plug-in Formulas**
  - GMAT may give you a strange formula and ask you for the result by giving you the values of all variables. **These problems are just a matter of careful computation.**

Quantitative Review
Algebra – Formulas

- **Formulas**
  - **Strange symbol formulas**
  - The symbol is irrelevant. Just follow each step of the procedure.
  - Example: if $x @ y = x^2 + y^2 - xy$, what is $3 @ 4$?
  - **Just replace: $3^2 + 4^2 - 3.4 = 25 – 12 = 13$**
  - Example: x¥ is defined as the product of all integers smaller than x but greater than zero. **This equals (x-1)!**
  - **Formulas with unspecified amounts**
  - Example: if the side of a cube is decreased by 2/3, by what percentage will the volume decrease?
  - **For those problems, pick numbers. Since $3^3 = 27$ and $1^3 = 1$, the percentage of decrease is 26/27, which is 96,3%**

Quantitative Review
Algebra – Formulas

– Sequence Formulas

- You must be given the rule in order to find a number in a sequence. Having two terms of the sequence is not enough.
- Linear sequences are also called arithmetic sequences. In those, the difference between two terms is constant. **$S = kn + x$**
- Exponential sequences: **$S = x(k^n)$**
- **If you are given that the first two terms of a sequence are 20 and 200, you know that $k = 200/20 = 10$. So, replacing: $S = x.10^n$. Now, you can find x: $20 = x.10^1$, so $x = 2$, and the sequence is 2.10^n**
- Other sequence problems: If each number in a sequence is 3 more than the previous and 6th number is 32, what is the 100th?
- We know that we have 94 jumps of 3 between the 6th and the 100th, so the answer is: **$94.3 + 32$**
- Sequences and patterns: If $S_n = 3^n$, what is the units digit for S_{65}?
- **There is a pattern: $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$. So, the units digit for 365 will be 3 again (64 is a multiple of 4)**

Quantitative Review
Algebra – Functions

- **Functions**
  - **Compound Functions**
  - **If $f(x) = x^3 + \sqrt{x}$ and $g(x) = 4x – 3$, what is $f(g(3))$?**
  - First, solve g(3): 4.3 – 3 = 9. Now, plug-in 9 on f(x): $9^3 + \sqrt{9} = 729 + 3$
  - **If $f(x) = x^3 + 1$ and $g(x) = 2x$, for what value $f(g(x)) = g(f(x))$?**
  - $f(2x) = g(x^3 + 1) = (2x)^3 + 1 = 2(x^3 + 1) = 8x^3 + 1 = 2x^3 + 2$ → **$x^3 = 1/6$**
  - **Proportional and inversely proportional functions**
  - For direct proportionality, set up ratios for the "before case" and the "after case" and equal them (regra de 3). **The proportionality is defined by $y = kx$, where k is the proportionality constant.**
  - For inverse proportionality, set up products for the "before" and "after" cases and equal them. The inverse prop. formula is **$y = k/x$**
  - Example: amount of current and resistance are inversely proportional. If current was 4 amperes and resistance is cut to one-third the original, what will be the current? **$x_1y_1 = x_2y_2$ – 3.4 = 1.x → x = 12**

- **Linear Growth or decay**
  - **They are defined by the function <u>y = mx + b</u>. m is the constant rate at which the quantity grows. b is the quantity at time zero.**
  - Example: a baby weighs 9 pounds at birth and gains 1.2 pounds per month. The function of his weight is: **<u>w = 1.2t + 9</u>** (t = nº of months)
- **Inequalities**
  - If you multiply or divide an inequality by a negative number, you have to flip the inequality sign.
  - **You cannot multiply or divide an inequality by a variable, unless you know the sign of the variable. The reason is you wouldn't be able to know whether you have to change the inequality sign.**
- **Combining Inequalities**
  - **Example: $x > 8$, $x < 17$, $x + 5 < 19$**
  - **(1) Solve any inequality that needs to be solved: $x + 5 < 19 = x < 14$**

Quantitative Review
Algebra – Inequalities

- **Combining Inequalities**
  - **(2) Make all the inequality symbols point in the same direction:**
    - **8 < x**
    - **x < 17**
    - **x < 14**
  - **(3) Eliminate the less limiting inequalities:**
    - **x < 14 is more limiting than x < 17, so ignore x < 17**
    - **The final answer is: 8 < x < 14**
- **Manipulating compound inequalities**
  - **If you perform an operation on a compound inequality, be sure you do it on every term.**
- **Combining inequalities**
  - **First, make all the inequality signs face the same direction. Them, add them up.**

Quantitative Review
Algebra – Inequalities

- **Combining Inequalities**
  - Example: Is a + 2b < c + 2d?
    - (1) a < c (2) d > b
  - a < c
  - \+ b < d
  - \+ b < d
  - = a + 2b < c + 2d
  - **You can always add inequalities. However, you can only multiply inequalities if both sides or both inequalities are positive. We can never subtract or divide inequalities!!**
- **Using extreme value: LT and GT**
  - **If 2h + 4 < 8 and g + 3h = 15, what is the possible range for g?**
    - **Since h<2, let's say h = LT2 (Less Than 2).**
    - **g + 3.LT2 = 15 → g = 15 – LT6**
    - **g = Greater than 9 → g > 9**

– Inequalities and Extreme Values

Operation	Example	Procedure
Addition	8 + LT2	LT10
Subtraction	8 – LT2 *	GT6
Multiplication	a) 8 . LT2 b) -7 . LT2 *	LT16 GT14
Division	8 / LT2	GT4
Multiply 2 extreme values	LT8 . LT2	LT16, but only if both are positive

- Take care with the ones with *

– Optimization problems

- On problems involving minimization or maximization, focus on the largest and smallest possible values of each variables.

Quantitative Review
Algebra – Inequalities

- Optimization problems
  - If $x \geq 4 + (z + 1)^2$, what is the minimum value for x?
  - **You have to recognize that a square value is minimized when set to zero.**
- **Testing Inequality cases**
  - Is $bd < 0$?
    - (1) $bc < 0$ (2) $cd > 0$

b	c	d	bc<0?	cd>0?	bd?
+	-	-	YES	YES	-
-	+	+	YES	YES	-

  - bd must be < 0 , as b and d will have different signs
  - **Whenever you need to proceed with a positive/negative analysis, building a table like the one above may be a good idea.**

Quantitative Review
Algebra – Inequalities

– Testing Inequality cases

- **It is important to know the following:**

Statement	Implication
$xy > 0$	X and y are <u>both positive or both negative</u>
$xy < 0$	X and Y have different signs
$x^2 - x < 0$	$x^2 < x$, so $0 < x < 1$ (x is a proper fraction)

– Inequalities and absolute value

- You should interpret some inequality problems with absolute value expressions as a range on the number line.
- **For more complicated problems, like $|x + 2| < 5$, a good method is to shift the entire graph down by 2. The center point will then change from 0 to 2**
- **<u>Standard formula: when $|x + b| < c$, center point is –b, and "less than" symbol tells us x is less than c units away from -b</u>**

Quantitative Review
Algebra – Inequalities

- **Square-Rooting inequalities**
  - If $x^2 < 4$, then $|x| < 2$. This means $x > -2$ and $x < 2$. So, $-2 < x < 2$.
  - You can only square-root an inequality if you know x is not negative
- **Inequalities Recap: Pg 99**

Do's	Don'ts
Do think about inequalities as ranges on a number line	Don't forget to flip the inequality sign if you multiply or divide by a negative number
Do treat inequalities like equations when adding or subtracting terms	Don't multiply or divide an inequality by a variable unless you know the variable's sign
Do line up multiply inequalities	Don't forget to take the most restrictive ineq.
Do add inequalities together when necessary	Don't forget to perform operations on all expr.
Do use extreme values to solve range problems	Don't subtract one inequality from another.
Do draw a number line if necessary	<u>Don't forget about the negative case</u>
Do set terms with even exponents equal to zero	Many pos/neg problems are disguised as ineq.

Quantitative Review
Algebra – VICs

- **VIC Problems**
  - Picking number to solve VICs:
    - (1) Never pick 0 or 1
    - (2) Make sure all the numbers you pick are different
    - (3) Pick Small numbers
    - (4) Try to pick prime numbers
    - (5) Don't pick numbers that appear as a coefficient in several answer choices
  - Direct Algebra
    - Break the problem down into manageable parts
    - Write every step, as it may be difficult to verify the answer
  - **Common Translations:**
  - **"y percent less than z" = z – yz/100**

Quantitative Review
Algebra – VICs

- **VIC Problems – Common translations**
  - **X is what percent of y, in terms of x and y?**
  - Answer: x = w/100 . y → x = wy/100 → **100x/y = w**
  - **If Cecil reads T pages per minute, how many hours does it take for her to read 500 pages?**
  - Answer: 500/60T
  - **X percent of Y percent of Z is decreased by Y percent. What is the result?**
  - Answer: XYZ/10,000 – (Y/100)*(XYZ/10,000) = (100XYZ – XY^2Z) / 1,000,000

– Equations – Advanced

- **Complex Absolute value equations:** If the equation contains only one variable, use algebra. If it contains 2 variables, go conceptual (positive/negative analysis).
- Example: $|x - 2| = |2x - 3|$
- You have to test 2 scenarios: the first with both equations positive, and the second <u>with one of them negative</u>. After finding the solutions, **you have to check all the answers.**
- **Multiplying or dividing two equations:**
  - If $xy^2 = 96$ and $1/xy = 1/24$, what is y?
  - We can easily solve by multiplying both equations: $xy^2 \,.\, 1/xy = 96 \,.\, 1/24$: $y = -4$
- **If you see similar/inverse equations, dividing/multiplying them may be a good idea to cancel variables.**

Quantitative Review
Algebra – Advanced Equations

– Advanced factoring and distributing

Distributed Form	Factored Form
$x^2 + x$	$x(x + 1)$
$X^5 – x^3$	$X^3(x^2 - 1) = x^3(x + 1)(x – 1)$
$6^5 – 6^3$	$6^3(6^2 - 1) = 35.6^3$
$4^8 + 4^9 + 4^{10}$	$4^8 + (1 + 4 + 4^2) = 21.4^8$
$p^3 - p$	$p(p^2 - 1) = p(p – 1)(p + 1)$
$a^b + a^{b+1}$	$a^b(1 + a)$
$m^n – m^{n-1}$	$m^n(1 – m^{-1}) = m^{n-1} (m-1)$
$5^5 - 5^4$	$5^5(1 – 1/5) = 5^4(5-1)$
$xw + yw + zx + zy$	$(w + z) (x + y) = w(x + y) + z(x + y)$

- **Be able to recognize any of the above and quickly manipulate it in both directions.**

Quantitative Review
Algebra – Formulas and Functions Advanced

- **Formulas and Functions – Advanced**
  - **Recursive formulas for sequences**
  - A recursive formula looks like this: $\mathbf{A_n = A_{n-1} + 9}$
  - **To solve a recursive sequence, you need to be given the recursive rule and also the value of one of the items.**
  - 1) Linear sequence: $S_1 = k + x$ → x is the difference between two terms, and k is the value of the sequence for S_0.
  - 2) Exponential sequence: $S_n = S_{n-1}k$ →You can find k by having 2 terms of the sequence.
  - **Exponential growth**
    - **The formula for exponential growth is: $Y_{(t)} = Y_0.k^t$ – y0 is the quantity when t = 0.**
  - **Simmetry**
    - **GMAT has difficult symmetry problem, such as:**
    - **For which of the following f(x) = f(1/x) ?**

- **Simmetry**
  - For those problems, you can either pick each alternative and substitute for the 2 variables (here, x and 1/x) or pick numbers. Normally, picking numbers is easier.
  - The same is valid for problems like: for which of the following f(x-y) does not equal f(x) – f(y)? For which of the following f(a(b+c)) = f(ab) + f(ac)?
- Optimization problems
  - **Maximizing linear functions**: since linear functions either rise or fall continuously, **the max/min points occur at boundaries**: at smallest or largest possible x as given in the problem.
  - **Maximizing quadratics**: The key is to **make the squared expression equal to 0**. Whatever value of x makes the squared expression equal 0 is the value of x that maximizes/minimizes the function

– Inequalities – Advanced

- **You cannot multiply or divide an inequality by a variable unless you know the sign of the variable!!**
- **Reciprocals of inequalities: <u>If we do not know the sign of both x and y, we cannot take reciprocals</u>**
- If x<y, then:
  - **1/x > 1/y when both x and y are positive**
  - **1/x > 1/y when both x and y are negative**
  - **1/x < 1/y when x is negative and y is positive**
- If ab < 0 and a>b, which is true?
  - (1) a>0 (2) b > 0 (3) 1/a > 1/b
- A good way to solve those problems is with a positive/negative analysis. We will see that 1 and 3 are true:

a	b	ab
+	-	-
-	-	+

- **Squaring inequalities**
  - **You cannot square both sides unless you know the signs of both sides.**
  - **If <u>both sides are negative</u>, the <u>inequality sign will flip</u> when you square.**
  - **If <u>both sides are positive</u>, the <u>sign will remain</u>.**
  - **<u>If one side of the inequality is positive and the other is negative, you cannot square!</u>**

– Word Translations

- **Algebraic Translation: Translating correctly**
  - A is half the size of B: A = 1/2B
  - A is 5 less than B: A = B – 5
  - Jane bought twice as many apples as bananas: A = 2B
  - **P is X percent of Q: P = xq/100 OR P/Q = x/100**
  - Pay $10 for the first 2 cds and 7 for additional CD: T = 10.2 + 7.(n-2)
  - ***** 1 year ago, Larry was 4 times older than John: L – 1 = 4J – 4 *****
- **Be ready to use expressions such as (n – x), where x will be the number of units with a different price**
- **Using Charts**
  - 8 years ago, George was half as old as Sarah. Sarah is now 20 years older than George. How old will be George in 10 years from now?

	8 yrs ago	Now	10 yrs
George	G – 8	G	?
Sarah	G + 12	G + 20	?

After building this table, it is easy to solve:

2 .(G – 8) = G + 20

2G – 16 = G + 12, G = 28

- **Word Translations**
  - **Hidden Constraints**
  - When the object of the problem has a physical restriction of being divided, such as votes, cards, pencils, fruits, you must realize the problem has told you the variable is an integer.
    - **Example: Jessica bought x erasers for 0.23 each and y pencils for 0.11 each. She has spent 1.70. What number of pencils did she buy?**
    - **23x + 11y = 170 -- 11y = 170 – 23x -- <u>y = (170 – 23x)/11</u>**
    - **Since the number of pencils is an integer, <u>y = (170 – 23x)/11</u> must be an integer as well. Test the values for x to find the answer.**
  - GMAT can also hide **positive constraints.** Sides of a square, number of votes, etc will always be positive. When you have a positive constraint, you can:
    - **Eliminate negative solutions from a quadratic function**
    - **Multiply or divide an inequality by a variable**
    - **Cross-multiply inequalities: $x/y < y/x \rightarrow x^2 < y^2$**
    - **Change an inequality sign for reciprocals: $x<y$; $1/x > 1/y$**

Quantitative Review
Word Translations – Rates & Work

- **Word Translations**
  - **Positive Constraints (cont.)**
    - **Multiply inequalities (but not divide them)**
    - **Square/unsquare an inequality: $x < y$; $x^2 < y^2$. $x < y$; $\sqrt{x} < \sqrt{y}$**
- **Rates and Work**
  - Rate x Time = Distance or Rate x Time = Work
  - RTD Chart: R x T = D, or R = D/T, or any other equation
  - **Always express rates as "distance over time". If it takes 4 seconds for an elevator to go up one floor, the rate is 0.25 floors / second.**
  - **Translations:**
    - **Train A is travelling at twice the speed of train B: A = 2r, B = r.**
    - **Wendy walks 1 mph slower than Maurice: W = r-1, M = r**
    - **Car A and car B are driving toward each other: Car a = r_1, Car b = r_2 – Shrinking distance = $r_1 + r_2$ (if they are driving away, this is growing distance)**
    - **Car A is chasing car B and catching up: a < b, and shrinking distance = a – b**

– Rates and Work - Translations

– **Sue and Sarah left the office at the same time, but Sue arrived 1h before: Sue's time = t-1, Sarah's time = t**

- **Table Samples (more examples on pg):**

– **Car A and Car B start driving toward each other at the same time:**

	Rate x	Time =	Distance
Car A	a	t	A's distance
Car B	b	t	B's distance
Total	a+b	t	Total distance
	ADD	KEEP	ADD

– **Jan drove from home to work. If she drove 10mph faster…**

Trip	Rate	Time	Distance
Actual	r	t	d
Hypothetical	r + 10	?	d
		VARIABLE	SAME

- **Rates and Work - Translations**
  - **Average Rate = Total Distance / Total time**
  - **Work Problems**
    - Normally, you will have to convert the rate. This is the only difference to the RTD problems. **Example: Oscar can perform one surgery in 1.5 hours. You have to convert it to: Oscar can perform 2/3 surgeries per hour. This is the actual rate.**
    - **If two or more agents are performing simultaneous work, add the work rates. (Typical problems are: "Machine A, Machine B").**
    - **Exepction: one agent undoes the other's work, like a pump putting water into a tank and another drawing water out. Rate will be a – b.**
    - Example: Larry can wash a car in 1 hour, Moe can wash it in 2 hours, and Curly washes it in 4 hours. How long does it take for them to wash 1 car?
    - **Answer: Rate is 1/1 + 1/2 + 1/4 = 7/4 cars/ hour. To find the rate for 1 car, just take the reciprocal: 4/7 hours to wash 1 car (34 minutes)**
  - **Population problems**
    - **In such problems, normally a population <u>increases by a common factor</u> every time period. You can build a population table to avoid computation errors.**

– Rates and Work - Population problems

– **Example: the population of a bacteria triples every 10 minutes. If the population was 100 20 minutes ago, when will it reach 24,000?**

Time Elapsed	Population
20 mins ago	100
10 mins ago	300
Now	900
10 mins after	2,700
20 mins after	8,100
30 mins after	24,300

This population table can be very useful to avoid mistakes doing the right computation but forgetting that the population right now is 900 and not 100, which was 20 minutes ago.

Population problems are just an exponential sequence since the bacterias grow by a <u>constant factor.</u> If something grows by a <u>constant amount</u>, you have a linear sequence.

• Ratios

– The relationship ratios express is division. A 3:4 rate is the same as a decimal 0.75.

– The order of the ratio has to be respected always.

– Rates and work – Advanced

- Advanced rates and work problems involve either more complicated language to translate, more complicated algebra to solve or trickier Data Sufficiency logic to follow
- **Be ready to break rate or work problems into natural stages. Likewise, combine workers or travelers into a single row when they work or travel together.**
  - **Example: Liam is pulled over for speeding just as he is arriving at work. He explains that he could not afford to be late today, and has arrived at work only 5 minutes before he is to start. The officer explains that if he had driven 5mph slower for his whole commute, he would have arrived on time. If his commute is 30 miles, how fast was he actually driving?**

	Rate	Time	Distance
Actual	r + 5	30/(r+5)	30
Hypothetical	r	30/r	30

  - **Since the slower trip takes 4 minutes more, $30/r = 30/(r+5) + 1/15 \rightarrow 30/r = (r + 455)/(15(r+5)) \rightarrow 450r + 2,250 = r^2 + 455r \rightarrow r^2 + 5r - 2,250 = 0 \rightarrow r = 45$**

– Equations for Exponential Growth or Decay

- Problems like: How long does it take to double the population?
- **Formula: <u>Population = $S.2^{t/I}$</u>, where S is the starting value, t is the time and I is the Interval, or the amount of time given for the quantity to double.**
- **If the quantity triples: <u>Population = $S.3^{t/I}$.</u>**
- **If the quantity is cut by half: <u>Population = $S.(0.5)^{t/I}$</u>**
  - Example: If a population of rabbits double every 7 months and the starting population is 100, what will be the population in 21 months?
  - **Answer: Population = $100 . 2^{21/7}$, = 100.2^3 = 800**

Quantitative Review
Word Translations - Ratios

- **Ratios: The unknown multiplier**
  - This technique can be used for complicated problems, such as: **the ratio of men to women is 3:4. There are 56 people in the class. How many are men?**
  - **Introducing the unknown multiplier: 3x + 4x = 56. x = 8, so we have 24 men.**
  - **You can use this technique for 3 ratios as well.**
- **Multiple ratios: Make a common Term**
  - **If C:A = 3:2, and C:L = 5:4, what is A:L?**
  - **C : A : L → C : A : L**
  - **3 : 2 : → Multiply by 5 → 15 : 10 :**
  - **5 : : 4 → Multiply by 3 → 15 : : 12**
  - **Once you have converted C to make both ratios equal (15), you can combine the ratios. Combined Ratio C:A:L = 15:10:12, and ratio A:L = 10:12 or 5:6**
- **The combined ratio will normally be asked as "the least number" of a population. The least number is the sum of common minimum denominator. In the situation above, the least number is 15 + 12 + 10 = 37.**

Quantitative Review
Word Translations – Ratios and Combinatorics

- **Productivity ratio**
  - For productivity ratios, <u>you should invert the time</u>. Example: If Machine A works for 80 minutes to produce X and has a productivity ratio of 4:5 to Machine B, how long does machine B take to produce X?
  - **5/4 = x/1/80 → 5.(1/80) = 4x → 4x = 1/16 → x = 1/64, so machine B takes 64 minutes to produce X.**
- **Combinatorics**
  - **Fundamental counting principle: if you must make a number of separate decisions, then <u>multiply the number of ways to make each individual decision</u> to find the number of ways to make all decisions together.**
    - **Example: If you have 4 types of bread, 3 types of cheese and 2 types of ham and wish to make a sandwich, you can make it in <u>4.3.2 = 24 different ways</u>.**
  - **For problems where some choices are restricted and/or affect other choices, choose most restricted options first (slot method):**
    - **Example: you must insert a 5-digit lock code, but the first and last numbers need to be odd, and no repetition is allowed. How many codes are possible?**

- **Combinatorics**
  - **Slot method:**
    - **1) Set up the slots: __ __ __ __ __**
    - **2) Fill the restricted: 5 __ __ __ 4**
    - **3) Fill the remaining: 5 . 8 . 7 . 6 . 4 = 6,720**
    - **The number of codes possible is 6,720**
  - **Combinations x Permutations**
    - You have to be ready to see the difference between the 2 statements below:
    - If seven people are going to sit in 3 seats (with 4 left standing), how many different seating arrangements can we have?
    - If three of seven standby passengers are to be selected for a flight, how many different combinations can we have?
    - **In the first problem, order matters. So, it is a permutation, and the formula is: <u>n! / (n – r)!</u>, where r! is the number of chosen items from a pool. For the second problem, all it matters is if the passenger is flying or not. Since the order of chosen passengers doesn't matter, the problem is a combination, and the formula is: <u>n! / (n-r)! r!</u>**

Quantitative Review
Word Translations – Combinatorics

- **Combinatorics**
  - **Permutation formula: <u>n! / (n – r)!</u>**
  - **Combination formula: <u>n! / (n-r)! r!</u>**
- **Multiple arrangements**
  - **In a group with 12 seniors and 11 juniors, you need to pick 3 seniors and 2 juniors. In how many different ways can you do that?**
  - **For these problems, you have to calculate the arrangements separately and then multiply both. Answer: 220 . 55 = 12,100**
- **Arrangements with constraints**
  - **When the problem presents you a constraint, such like "Jan won't sit next to Marcia", <u>you should first calculate the arrangement without considering the constraint</u> and then using a method for the constraint.**
  - Example 1: Greg, Mary, Pete, Jan, Bob and Cindy are to sit in 6 adjacent seats, but Mary and Jan won't sit next to each other. How many combinations?
  - **Without the constraint, you have 6!, which is 720. Now, use the <u>glue method:</u> consider that an and Marcia are one person. The combination would then be 5!, which is 120. You have to multiply this by 2, as Marcia could sit next to Jan OR Jan could sit next to Marcia. So, the number of total restrictions is 120 . 2 = 240, and the number of combs is 720 – 240 = 480**

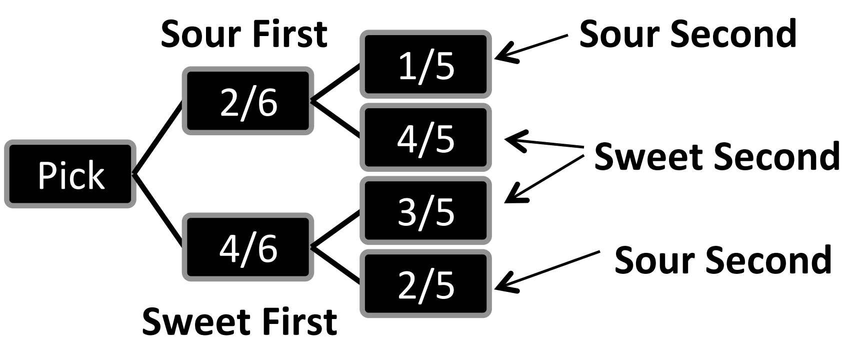
Quantitative Review
Word Translations –Combinatorics

- **Arrangements with constraints**
  - **Example 2: Gnomes and Dwarfs**
  - **Example 3**: Six friends are to sit in a line, but Joey doesn't want to sit behind Paul. What is the total number of possibilities for this to happen?
  - **The total number of arrangements is 6!, which is 720. Now, you have to realize that Joey would be sitting behind Paul <u>in half of the possibilities.</u> This would happen with any friend, as half of the combinations would have one of them sitting behind, and the other combination would have him sitting in front. So, the answer is 360, as half of the combinations is restricted.**

– Probability

- <u>Probability = Nº of desired results / Nº of possible outcomes</u>
- More than one event: "and" vs "or"
  - And means <u>multiply probabilities</u>. Or means <u>sum probabilities.</u>
  - Example: Molly needs to roll a 1 in a die and she has 3 tries. What is the probability that she will succeed?
  - Probability to roll a 1 at the first try: 1/6
  - Probability to roll a 1 at the second try: 5/6 * 1/6 = 5/36
  - Probability to roll a 1 at the third try: 5/6 * 5/6 * 1/6 = 25/216
  - Since she can succeed with one OR two OR three tries, we sum it: 91/216
- When you have an OR problem where events cannot happen together, just add the probabilities (as above). When the events <u>can</u> happen together, use the formula: <u>P(a or b) = P(a) + P(b) – P(a AND b)</u>
  - Example: What is the probability of getting either a 3 in a die or a head in a coin?
  - <u>P(a) = 1/6. P(b) = 1/2. P(a AND b) = 1/12. P(a or b) = 1/6 + 1/2 – 1/12 = 7/12</u>

- **Probability**
  - **The 1 – x probability trick**
    - If on a GMAT problem it's too complicated to calculate "success", you can calculate the probability that success does not happen. The probability of success will be 1 – x, where x is the failure probability.
  - **The domino effect (actions that affect the next probability)**
    - In a box with 10 blocks (3 are red) what is the probability of picking a red block on <u>each</u> of your first 2 tries if you discard a block after picking it?
    - Answer: 3/10 . 2/9 = 6/90 = 1/15
  - **Probability trees**
    - Renee has a bag of 6 candies, 4 of which are sweet and 2 are sour. Jack picks two candies at random. What is the probability that he got exactly 1 sour?
    -
    -



Quantitative Review
Word Translations – Probability and Statistics

- **Probability trees**
  - **The scenarios that interest us are: Sour first and sweet second (2/6 * 4/5 = 8/30) and Sweet First and Sour second (4/6 * 2/5 = 8/30). The probability of getting exactly one sour candy by picking 2 candies is: 8/30 + 8/30 = 16/30**
- **Statistics**
  - **Evenly spaced sets**
    - **The average of an evenly spaced set is: (FIRST TERM + LAST TERM) /2**
  - **Weighted averages**
    - If 94 weighs 40%, 88 weighs 30%, 98 weighs 20% and 85 weighs 10%, what is the average?
    - Answer: 94 * 0.4 + 88 * 0.3 + 98 * 0.2 + 85 * 0.1 = 92.1
    - **Properties of weighted averages:**
    - **You do not need concrete weight values. Having the ratio is enough.**
    - **A weighted average will fall closer to whichever value is weighted more heavily. If the computation is difficult, you can try to eliminate alternatives.**

Quantitative Review
Word Translations – Statistics

- ## Median
  - Given the set {x,x,y,y,y,y}, where y>x. Is the median bigger than the mean?
  - Mean = (2x + 4y) / 6
  - Median = y
  - y > (2x + 4y) / 6? → 6y > 4y + 2x? → 2y > 2x? → y > x, so median is bigger
- ## Standard Deviation
  - **If each data point in a set is increased by a number, the standard deviation won't be affected. <u>If the whole set is increased by a factor of 7 ("multiply by 7"), the standard deviation will increase by a factor of 7</u>.**
- ## Overlapping sets
  - **The double-set matrix**
    - Columns must represent opposite information from each other, and the same is required for the rows. For example: if you have students that study either spanish or english, columns can contain "spanish/not spanish", while rows can contain "french/not french".

– Overlapping sets

- **Example: Of 30 integers, 15 are in set A and 22 are in set B. If 8 are In both, how many are out of both?**

	A	Not A	Total
B	8	14	22
Not B	7	<u>**1**</u>	8
Total	15	15	30

- **You were only given the numbers in black, but it is very easy to find any other number in the matrix. You have to focus in the following: If you are given that <u>x</u> belong to group A and the total of members is <u>y</u>, <u>then the number of members who do not belong to group A are y – x.</u> This is mandatory to solve most of the problems.**
- **Example:**

– Overlapping sets and algebraic representation

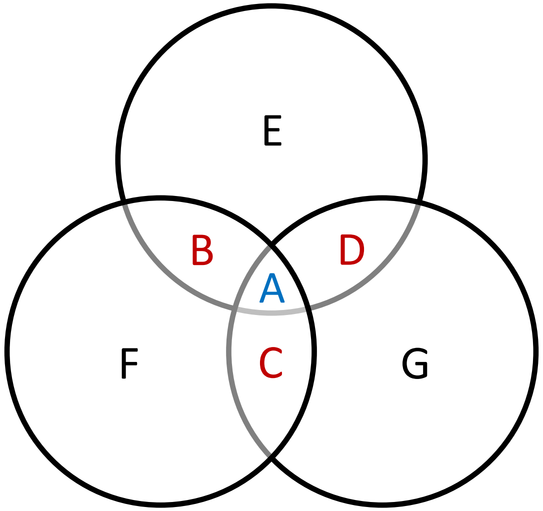
- **Example: 10% of the children have been good during the year, but do not celebrate xmas. 50% of the ones that celeb. xmas have been good. If 40% of all children have been good, how much % of children do not celebrate xmas?**

	Good	Not good	Total
Xmas	0.5x	0.5x	x
No Xmas	10		
Total	40		

- **You know that 50% of the ones that celebrate xmas have been good, so 50% haven't. If the number of children that celebrate xmas is x, then 0.5x have been good. <u>Looking at table we can easily see that 0.5x = 30</u>. So, x = 60. If 60% of the children celebrate xmas, 40% do not celebrate it.**
- For problems with 2 sets and 3 choices, you should also use the double-set matrix. For problems with 3 sets, use the venn diagram.

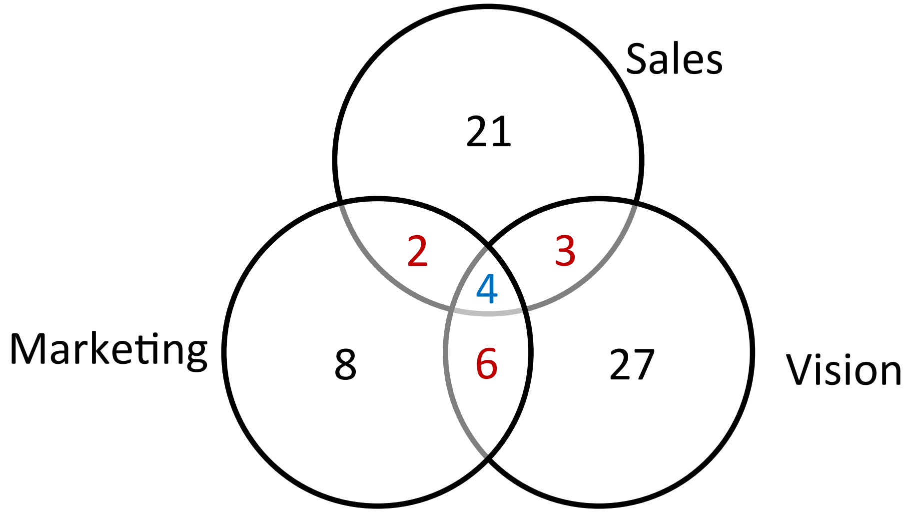
– Overlapping sets and the Venn Diagram

- A Venn Diagram has 7 sections: Section A, where the 3 circles overlap, sections B, C and D, where 2 circles overlap and sections E, F, G, where none overlap.
- **The main rule for a Venn Diagram is: <u>work from the inside out!!</u>**



– **Example: Workers are grouped by teams in a company. 20 workers are on marketing, 30 are on sales, 40 are on vision. 5 workers are both on marketing and sales, 6 workers are both on sales and vision, and 9 are both on marketing and vision. 4 are in all teams. How many workers in total?**

- **Overlapping sets and the Venn Diagram**
  - **Answer: 4 + 2 + 3 + 6 + 21 + 27 + 8 = 71 workers**



- **Minor problem types**
  - **The general approach for optimization, grouping and scheduling problems is to focus on extremes.**
  - **<u>Optimization problems</u>**
    - The guests of a banquet consumed 401 pounds of food. If no one consumed <u>more</u> than 2.5 pounds of food, what is the minimum number of guests?

Quantitative Review
Word Translations – Minor problem types

- **Optimization Problems**
  - **Answer: since 401 / 2.5 = 160.4 and no guests ate MORE than 2.5 pounds of food, the minimum number of guests will be 161. If no one ate LESS than 2.5 pounds, the MAXIMUM number of guests would be 160.**
- **Grouping Problems**
  - You need to group 1 person from division A, 2 from division B and 3 from division C. Division A has 20 people, division B has 30, and C has 40. What is the smallest number of people who will not be assigned to a group?
  - **A: 20/1 = 20 groups are possible**
  - **B: 30/2 = 15 groups are possible**
  - **C: 40/3 = 13 groups are possible.**
  - **The limiting factor is group C, as they have the smallest number of groups to be formed. Therefore, we can only form 13 groups, and this is when the smallest number of people will be left over: <u>12 people in total</u>.**
- **Scheduling**
  - **How many days does the standard warranty of product X last?**
  - **(1) Marck purchased in jan 97. Warranty didn't expire until march**
  - **(2) Santos purchased in May 94. Warranty expired in may.**

– Scheduling

- **Answer:** Statement (1) tells us the warranty lasts <u>from 29 days</u> (if the product was purchased by jan 31st and warranty finished by march 1st) <u>to 89 days</u> (if the product was purchased by jan 1st and warranty finished by march 31st). **<u>INSUFFICIENT</u>**
- Statement (2) tells us the warranty lasts 1 to 30 days . **<u>INSUFFICIENT</u>**
- **Both statements together are <u>INSUFFICIENT</u>, as the warranty could last 29 or 30 days.**

– Comb / Prop / Stats Advanced

- Jane needs to walk 3 blocks south and 3 blocks east. What is the probability that she will walk south for the first 2 blocks?
- **Answer: the total number of ways she can walk 6 blocks being 3 in one way and 3 in another way are: 6! / 3!3! = 20. If she will walk the first 2 blocks south, <u>use the reducing pool/glue technique</u>. Since you already know she has walked the first 2 blocks to south, she needs to walk 4 blocks, being one to south and 3 east. Number of possible way is 4! / 3!1! = 4. So, the probability she will walk south for the first 2 blocks is 4/20 = 1/5**

Quantitative Review
Word Translations – Comb / Prop / Stats Advanced

- **Combinations vs Permutations**
  - **If switching the elements in a chosen set creates a different set, it is a permutation. If not, it is a combination.**
- **Combinatorics and Domino Effect**
  - If you have a symmetrical problem, you can calculate the probability of one case and multiply by the number of cases.
    - **Example: A machine contains 7 blue, 5 green and 4 red gumballs. If machine dispenses 3 balls at random, what is the prob. of being 1 each color?**
    - **Answer: 7/16 * 5/15 * 4/14 = 1/24. This can happen in 6 different ways, as we have 3 balls to be taken out and there are 3 possible outcomes. So, you just find one probability (1/24) and multiply by 3!, which yields 1/4.**
- **Shortcuts for averages**
  - Advanced DS questions may give you enough information to know how much the average will change when a new data point is inserted <u>even if you don't know how to calculate the average itself</u> (so you won't be able to know the percent change on the mean)

- **Shortcuts for averages**
  - **To know how much the mean will change, you only need to know the difference between the new term and the mean, and also the number of terms**
  - **Residuals are the difference between a set's data points and its average. For any set, the residuals sum to zero.**
    - **Example: If the mean of the set {97, 100, 85, 90, 94, 80, 92, x} is 91, what is the value of x?**
    - Instead of doing all the calculation, you can find the residual of each term: +6, +9, -6, -1, +3, -11, +1. This sum yields +1, so x leaves a residual of -1 when compared to 91. **x = 90**.

- **Divisibility and Primes**
  - **Arithmetic rules:**
    - **The sum/difference/product of 2 integers is always an integer**
    - **The quotient of two integers is not always an integer**
    - **<u>An integer is said to be divisible by another if the result is also an integer</u>**
  - **Rules of divisibility by certain integers**
    - **<u>An integer is divisible by:</u>**
    - **2: if it is EVEN**
    - **3: if the sum of integer's digits is divisible by 3**
    - **4: if the integer is div. by 2 twice or if THE LAST TWO DIGITS are divisible by 4**
    - **5: If the integer ends in 5 or 0**
    - **6: if the integer is divisible BOTH by 2 and 3**
    - **8: if the integer is divisible by 2 three times or IF THE LAST 3 DIGITS are divisible by 8**
    - **9: if the sum of the integer digits is divisible by 9**
    - **10: if the integer ends in 0**
    - **11: sum of digits in odd places – sum of digits in even places either = 0 or divisible by 11.**

– Factors and Multiples

- Method 1: building table with factor pairs:
  - Example: How many factors does 72 have?
  - **We can see that 72 has 12 factors**
  - Any integer has a limited number of factors but infinite number of multiples.

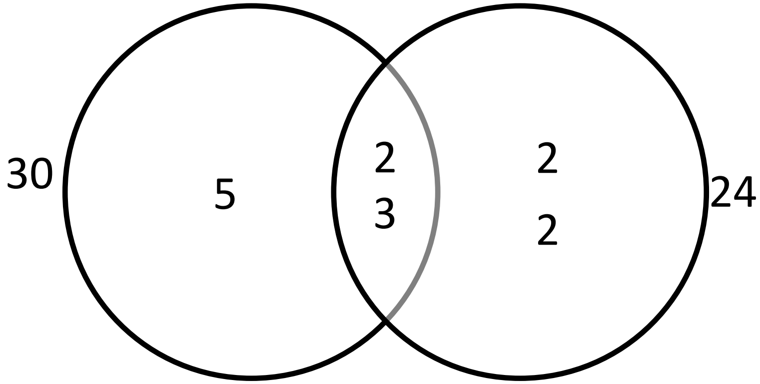
Small	Large
1	72
2	36
3	24
4	18
6	12
8	9

- **Method 2 (Recommended):**
- **<u>If a number has prime factorization $a^x \cdot b^y \cdot c^z$, where a, b and c are primes, then the number has (x+1)(y+1)(z+1) total factors.</u>**
  - Example: How many total factors does the number 2,000 have?
  - **With prime factorization we find that 2,000 = $2^4 \cdot 5^3$. So, 2,000 has 5.4 = 20 total factors**

- **Prime Factorization**
  - Use prime factorization to:
    - Determine whether a number is divisible by another
    - Determine the GCF / LCM of 2 or more numbers
  - The prime box
    - If n is divisible by 3, 7 and 11, what other numbers are divisors of n?
    - 3.7 = 21, 3.11 = 33, 7.11 = 77, 3.7.11 = 231. **If a number has the primes 2, 3 and 7 in its prime box, this number will also be divisible by any combination of them.**
  - **Finding GCF and LCM using Venn Diagrams**
    - **(1) Factor the numbers into primes**
    - **(2) create a venn diagram**
    - **(3) Place common factors in shared areas**
    - **(4) place the remaining factors in non-shared areas**
    - **<u>GCF is the product of primes in the overlapping region</u>**
    - **<u>LCM is the product of all primes in the diagram</u>**

Quantitative Review
Number Properties – Divisibility and primes

- **Finding GCF and LCM using Venn Diagrams**
  - **Example: Find GCF and LCM for 24 and 30:**



  - **GCF = 2 . 3 = 6 . LCM = 2 . 2 . 2 . 3 . 5 = 120**
  - **<u>If 2 numbers have no primes in common, their GCF is 1</u>**
- **Remainders**
  - **On simpler remainder problems, the best technique is to pick numbers.**
- **Advanced remainder techniques**
  - **If you need a number that leaves a remainder of R when divided by N, simply take any multiple of N and add R.**
  - **The remainder of any number MUST be non-negative and smaller than the divisor.**

Quantitative Review
Number Properties – Divisibility and primes

- **Advanced remainder techniques**
  - **You can add, subtract and multiply remainder, as long as you correct the final result by dividing the resultant number by the divisor.**
  - **Common GMAT problem:** If A/B yields 4.35, what could be the remainder?
  - (A) 13 (B) 14 (C) 15 (D) 16 (E) 17
  - **Answer:** First, convert .35 to a fraction: .35 = 35/100 = 7/20. Now, compare this fraction with B (the divisor on this problem):
  - **7/20 = R/B → From this equation, we know that the remainder <u>must be a multiple of 7.</u> The correct answer is 14.**

- **Odds and Evens**
  - **<u>Addition/subtraction</u>**: If you add/subtract 2 odds or 2 evens, the result will be <u>even</u>. If you add/subtract an odd with an even, result will be <u>odd</u>.
  - **<u>Multiplication</u>:** If any of the integers in a multiplication is even, result is <u>even</u>**.**
  - <u>If 2 even integers are multiplied, result is divisible by 4, and if 3 even integers are multiplied, result is divisible by 8</u>. If many even integers are multiplied, the result is higher and higher powers of 2, as 2 is the only prime integer.

- **Odds and Evens: Division**
  - An even divided by another even can yield an even, odd or non-integer result.
  - An even divided by an odd cannot yield an odd number (either even result or non-integer result)
  - An odd divided by an even will always yield a non-integer.
  - An odd divided by another odd cannot yield an even number.

- **Prime numbers**: Since 2 is the only even prime number, **the sum/difference of two primes can only result an odd number if one of them is 2. In this case, the product will be even**. For all other cases, the product will be odd and the sum/difference will be even. Conversely, knowing that the product of two prime numbers is even OR that the sum/difference is odd is **sufficient** to know that one of the prime numbers is 2.
  - **Example**: If x>1, what is x?
  - (1) There are x unique factors of x (2) x plus any prime larger than x is odd.
  - Statement (1) is sufficient as this property holds only for 1 and 2. Statement 2 only tells us that x is even, as it is not stated in the problem that x is prime.

Quantitative Review
Number Properties – Divisibility and primes + POS/NEG

- **Testing odd and even cases**
  - Example: If a, b and c are integers and a.b + c is odd, what is true?
  - I. a + c is odd
  - II. b + c is odd
  - III. a.b.c is even
  - **Either a and b are odd and c is even, or c is odd and a, b or both are even. Therefore, a + c can be odd or even, and the same happens for b + c. So, the only necessarily true statement is III.**
- **Positives and Negatives**
  - **When you multiply or divide a group of nonzero numbers, the result will be positive if you have <u>an even number of negative numbers</u>. If not, the result will be negative.**
  - **Example:** Is the product of all elements in set S negative?
  - **(1) All the elements in set S are negative**
  - **(2) There are five negative elements in S.**
  - This is a trap, as statement 2 seems sufficient, but it is not as we don't know whether there is a 0 on set S or not. Both statements together are sufficient.

Quantitative Review
Number Properties – Positive / Negative

- **Testing positive and negative cases**
  - **Example:** If ab > 0, which of the following is negative?
  - **(a)** a + b **(b)** |a| + b **(c)** b – a **(d)** a/b **(e)** –a/b
  - To solve these problems, build a positive/negative table and if necessary, pick numbers:

	a + b	|a| + b	b – a	a/b	-a/b
a = 3, b = 6	POS	POS	POS	POS	NEG
a = -3, b = -6	NEG	NEG	NEG	POS	NEG

  - **Picking 2 sets of numbers was sufficient to solve. Answer: E**

- **Consecutive Integers**
  - **Consecutive integer sets are a special type of evenly spaced sets, as the distance between each term is always 1.**
  - **Properties of evenly spaced sets:**
  - **<u>The arithmetic mean and the median are always equal</u>**
  - **Mean and median are equal to the <u>average of FIRST and LAST terms</u>**
  - **<u>The sum of all elements is equal to the mean times the number of items</u>**

Quantitative Review
Number Properties – Consecutive Integers

- **Consecutive Integers: Counting Integers**
  - **Example:** How many integers are there from 14 to 765, inclusive?
  - **Formula: Last Term – First Term + 1**
  - **Example 2:** How many even integers are there from 12 to 24?
  - **Formula: ((Last Term – First Term)/Increment) + 1**
  - **Example 3:** How many multiples of 7 are there from 100 to 150?
  - **First, find the first and last multiples of 7: The first is 105, and the last is 147. Now, apply the formula: (147 – 105)/7 + 1 = 7**
- **The sum of consecutive integers**
  - **Common on GMAT:** What is the sum of all integers from 20 to 100, inclusive?
  - (1) Count the number of terms: **100 – 20 + 1 = 81**
  - (2) Find the average: **(100 + 20)/2 = 60**
  - (3) Multiply the average by the number of terms: 60 * 81 = 4,860
- **Important properties**
  - The average of an odd number of consecutive integers <u>will be an integer</u>
  - The average of an even number of consecutive integers <u>will not be an integer</u>

Quantitative Review
Number Properties – Consecutive Integers And Exponents

- **Important properties of consecutive integers**
  - **Example:** Is k^2 odd?
  - (1) k-1 is divisible by 2
  - (2) The sum of k consecutive integers is divisible by k
  - **Answer:** Statement (1) tells us k is odd, so k^2 is also odd. **Sufficient**
  - Statement (2) tells us the average of a set with k consecutive integers is an integer, so k must be odd and k^2 is also odd. **Sufficient**. **Answer: D**
- **Product of consecutive integers and divisibility**
  - **The product of n consecutive integers is divisible by n!**
  - **The sum of n consecutive integers is divisible by n IF n is odd, and IS NOT divisible by n if n is even.**
- **Exponents**
  - **Even exponents hide the sign of their base. GMAT uses this trick constantly.**
  - **Fractional base: When the base is between 0 and 1, if the exponent increases, the value of the expression decreases.**

Quantitative Review
Number Properties – Exponents

- **Compound bases**
  - **<u>You can distribute the exponent to each number in the base and also decompose the base: $(2.5)^3 = 10^3 = 2^3.5^3$. It is essential to master this technique!!! This CANNOT be done to a sum.</u>**
- **The exponent**
  - **When multiplying 2 terms with the same base, add the exponents. When dividing, subtract the exponents.**
  - **Nested exponents: $(3^2)^3 = 3^6$**
  - **An expression with negative exponent is the reciprocal of what that expression would be with positive exponent.**
- **Rules of exponents**
  - **$(a/b)^x = a^x/b^x$**
  - **$3^3 + 3^3 + 3^3 = 3.3^3 = 3^4$**
  - **$a^x b^x = (ab)^x$**
  - **$x^{-a} = 1/x^a$**

- **Simplifying exponential expressions**
  - **Always try to simplify exponential expressions when they have the same base or the same exponent. When expressions with the same base are linked by a sum, you cannot simplify but you can factor the expression:**
  - **$7^2 + 7^3 = 7^2.(7 + 1) = 49.8$**
  - **Example:** what is the largest prime factor of $4^{21} + 4^{21} + 4^{23}$?
  - **Answer:** $4^{21} + 4^{21} + 4^{23} = 4^{21}(1 + 4 + 16) = 4^{21}.(21) = 4^{21}(3.7)$. **The largest prime factor of $4^{21} + 4^{21} + 4^{23}$ is 7**
- **Roots**
  - **Main difference between exponents: even roots have only one solution. √4 can only be 2, and not -2.**
  - Negative roots can only exist if they are odd: **$\sqrt[3]{-27}$ exists.**
- **Simplifying a root**
  - **GMAT very often tries to trick you by giving a root linked by addition where it is tempting to simplify the terms, for example: √(25 + 16). It is tempting to think that this will result into 5 + 4, but <u>you can only simplify roots when the terms inside/outside are linked by multiplication or division</u>**

Quantitative Review
Number Properties – Exponents

- **Simplifying roots**
  - **Examples of roots simplification:**
  - $\sqrt{25.16} = \sqrt{25} . \sqrt{16} = 5.4$
  - $\sqrt{50} . \sqrt{18} = \sqrt{50.18} = \sqrt{900} = 30$
  - $\sqrt{144:16} = \sqrt{144} : \sqrt{16} = 12/4 = 3$
  - **$\sqrt{25+16} = \sqrt{41}$ → you cannot simplify this one**
- **Knowing common roots**
  - **GMAT requires you to know all the perfect square roots from 1 to 30 and also the following imperfect rules:**
  - $\sqrt{2} \approx 1.4$
  - $\sqrt{3} \approx 1.7$
  - $\sqrt{5} \approx 2.25$
- **Order of operation: PEMDAS**
  - **PEMDAS = <u>P</u>arentheses, <u>E</u>xponents, <u>M</u>ultiplication & <u>D</u>ivision, <u>A</u>ddition & <u>S</u>ubtraction**
  - **Most common mistake with computations: not distributing the subtraction sign to all the terms. TAKE CARE.**
  - **Example: $(3x – 3) – (4x – 2) = 3x – 3 – 4x + 2$**

- **Divisibility & Primes advanced**
  - **There are 25 primes until 100. Try to know all of them. Besides 2 and 5, they all finish with 1, 3, 7 or 9. {2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97}**
  - **<u>There is an infinite number of primes</u>**
- **Divisibility and addition/subtraction**
  - **If you add or subtract multiples of an integer, the result is also a multiple.**
  - **<u>If you add/subtract a multiple of N to a non-multiple of N, result is non-multiple of N</u>**
  - **When you add two non-multiples of 2, the result is a multiple of 2**
- **Advanced GCF and LCM techniques**
  - **Finding GCF and LCM of 3 or more numbers:**
  - **(1) calculate the prime factors of each integer**
  - **(2) create a column for each prime factor found and a row for each integer**
  - **Raise each prime to a power, which is the number of times it appeared on that integer**

Quantitative Review
Number Properties – Divisibility & Primes advanced

- **Advanced GCF and LCM techniques**
  - **Example:** Find the GCF and LCM of 100, 140 and 250
  - Prime factors: **100** → 2,2,5,5. **140** → 2,2,5,7. **250** → 2,5,5,5

Integer	2	5	7
100	2^2	5^2	-
140	2^2	5^1	7^1
250	2^1	5^3	-

  - **GCF is the smallest count in any column: $2^1 \cdot 5^1 = 10$**
  - **LCM is the largest count in any column: $2^2 \cdot 5^3 \cdot 7^1 = 3{,}500$**
- **Three important properties of GCF and LCM:**
  - **(1) (GCF of m and n) . (LCM of m and n) = m.n**
  - **(2) GCF or m and n cannot be larger than the difference of m and n**
  - **(3) Consecutive multiples of n have a GCF of n. Example: the GCF of 84 and 88 is 4.**

- **Combinations of GCF or LCM**
  - **Very common on GMAT**
  - **Example:** Is z divisible by 6?
  - (1) the GCF of z and 12 is 3
  - (2) the GCF of z and 15 is 15
  - **If the GCF of z and 12 is 3, we know that z is divisible by 6 but not by 2. Therefore, z is not divisible by 6. (1) is SUFFICIENT**
  - **If the GCF of z and 15 is 15, we know that z is divisible by 3 and 5, but we do not know if z is divisible by 2 or not, so (2) is INSUFFICIENT. Answer: A**

  - **Example 2:** If the LCM of a and 12 is 36, what are the possible values for a?
  - **The prime factorization of 12 results into 2,2,3. The prime factorization of 36 results into 2,2,3,3. Therefore, a must have one non-shared 3 with 12 in its prime factorization, and no other non-shared prime. So, a could be:**
  - **<u>3.3 = 9</u>, OR <u>2.3.3 = 18</u>, OR <u>2.2.3.3 = 36</u>**
- **Counting total factors**
  - **If you are asked "how many different prime factors" a number or a product has, you don't need to consider how many times each prime is repeated.**

Quantitative Review
Number Properties – Divisibility & Primes advanced

- **Counting total factors**
  - **If you are asked <u>“how many total prime factors” or “what is the length”</u> of a number, then you need to count repeated prime factors as well.**
  - **If you are asked “how many total factors”, use the technique showed on slide 82.**
- **Perfect squares, cubes, etc**
  - **All perfect squares have an odd number of total factors. Conversely, any integer that has an odd number of total factors is a perfect square. This happens because this number will have a factor pair with 2 repeated numbers.**
  - **The prime factorization of perfect squares contains only even powers of primes. This rule extends to cubes and other powers.**

  - **Advanced question: if k^3 is divisible by 240, what is k's least possible value?**

Quantitative Review
Number Properties – Divisibility & Primes advanced

- **Counting total factors**
  - **Advanced question: if k^3 is divisible by 240, what is k's least possible value?**
  - **The prime factorization of k^3 results <u>AT LEAST</u> on 2,2,2,2,3,5. So, since k^3 = k.k.k, each k has at least two 2's, one 3 and one 5. So, k's least possible value is 2.2.3.5 = 60**
- **Factorials and divisibility**
  - **N! is multiple of all integers from 1 to N**
  - **10! + 11! Is a multiple of any integer from 1 to 10, as both terms include every integer from 1 to 10.**
- **Odd/Even, Pos/neg, Consec Integers Advanced**
  - **Property: odd integers leave a remainder of 1 when divided by 2**
  - **Example:** If x, y and z are integers and xyz is divisible by 8, is x even?
  - (1) xy is divisible by 4
  - (2) x, y and z are all not divisible by 4
  - **(1) is not sufficient, as y could be divisible by 4 and x could be odd.**
  - **(2) is SUFFICIENT, as if neither x, y and z are divisible by 4, all of them must be divisible by 2 in order to let xyz be divisible by 8. Answer: B**

Quantitative Review
Number Properties – Odd/Even, Pos/Neg Advanced

- **Representing Evens and Odds algebraically**
  - **Even numbers are multiple of 2, so an arbitrary even number can be written as 2n. <u>Odd numbers are one more or one less than multiples of 2, so they can be written as 2n+1 or 2n-1</u>**
  - **Advanced question:** What is the remainder of a/4?
  - (1) a is the square of an odd integer
  - (2) a is a multiple of 3
  - **Answer: if we square (2n+1) we have $4n^2 + 4n + 1$, so it is clear that any odd number, when squared and further divided by 4 will leave a remainder of 1. So, (1) is SUFFICIENT. Statement (2) is INSUFFICIENT, as a could be an even number, like 6 or 12.**
- **Absolute value**
  - **The absolute value $|x – y|$ must be interpreted as <u>the distance between x and y</u>. If you see the expression $|x – 3| < 4$, think that "the distance between x and 3 is less than 4 units".**
  - **<u>When you have 2 variables in absolute value equations, you should pick numbers to eliminate answer choices.</u>**

Quantitative Review
Number Properties – Odd/Even, Pos/Neg Advanced

- **Absolute value**
  - **Example:** $|x| - |y| = |x + y|$, and $xy \neq 0$. Which must be true?
  - **(A) $x - y > 0$ (B) $x - y < 0$ (C) $x + Y + 0$ (D) $xy > 0$ (E) $xy < 0$**
  - **First, picking $x = 3$ and $y = -2$, we eliminate B and D. Then, picking $x = -3$ and $y = 2$, we eliminate A and C. Hence, answer is E.**
- **Consecutive integers and divisibility**
  - **Advanced question: if $x^3 - x = p$ and x is odd, is p divisible by 24?**
  - **Factoring $x^3 - x$: $x(x^2 - 1)$. Factoring $(x^2 - 1)$: $(x + 1)(x - 1)$. So, the expression is $x(x+1)(x-1)$, or <u>$(x-1)x(x+1)$</u>, which denotes a consecutive integers expression. If x is odd, x-1 and x+1 are even. So, p is at least divisible by 4. Also, one of them will at least be divisible by 2 twice, so p is at least divisible by 8. Finally, the product of any 3 consecutive numbers is divisible by 3, so p is divisible by 24.**
- **Exponents and roots Advanced**
  - **Whenever two or more terms with exponents are added or subtracted, consider factoring them.**
  - **Whenever possible, break the bases into prime factors and distribute the exponents.**

Quantitative Review
Number Properties – Exponents and Roots Advanced

- **Using conjugates to rationalize denominators**
  - **4 / √2 = 4/ √2 * √2/ √2 = 4 √2/2 = 2√2**
  - **4 / (3 - √2) = 4 / (3 - √2) * (3 + √2) / (3 + √2) = (12 + 4 √2)/(9 + 3 √2 - 3 √2 – 2) = <u>(12 + 4 √2)/7</u>**
  - **<u>To rationalize conjugates like a + √b, multiply by a - √b.</u>**
- **Important exponents properties**
  - **<u>If x, z and y are integers and $2^x.5^x = 2^{2y}.5^z$, then x = 2y and x = z, as each side of the equation must have the same number of prime factors.</u>**

Quantitative Review
FDPs - Decimals

- **Fractions, Decimals, Percents**
  - **Rounding to the nearest place value:**
  - **You must memorize tenths, hundredths, thousandths and ten thousandths.**
  - Round looking at the right digit number. If it is a 5 or greater, round it up. If not, keep the original value.
  - **Decimal operations: what is 10 – 0.063?**
  - **The key is to add zeroes to the bigger number to avoid confusions: 10.000 – 0.063. Now they have the same length and it is easier to do the subtraction.**
  - **Multiplication: ignore decimal point until the end. Multiply as if numbers were whole. Then, count the number of digits to the right of the decimals in both numbers and apply it to the product.**
  - **Example:** 1.4 . 0.02 → 14 x 2 = 28, and we had 1 decimal on 1.4 and 2 decimals on 0.02, so number is 0.028
  - **The same rules apply for division.**

- **Decimals on Powers and roots**
  - Always use the integer. If necessary, convert the decimal into powers of ten.
  - Example: $(0.5)^3$ → $0.5 = 5.10^{-1}$ → $5^3.10^{-3}$. = $125 . 10^{-3} = 0.125$

- **Decimals on Powers and roots**
  - Always use the integer. If necessary, convert the decimal into powers of ten.
  - Example 2: $\sqrt[3]{0.000027} = (27 \,.\, 10^{-6})^{1/3} = 27^{1/3} \,.\, (10^{-6})^{1/3} = 3.10^{-2} = 0.03$
  - **This technique is very important and can help you answering questions that seem complicated but become very easy when we use this.**

  - **Shortcut: the number of decimal places in the result of a x-numbered decimal is x times the number of decimals of the original decimal:**
  - **$(0.04)^3 = 0.04 * 0.04 * 0.04 = 0.000064$ → 2 decimals powered to 3 yield 6 decimals.**
  - **$\sqrt[3]{0.000000008} = 0.002$. A number with 9 decimals cube-rooted yields a number with 3 decimals.**

- **Fractions**
  - **If you add the same number to both the numerator and denominator of a fraction, the fraction gets closer to 1, regardless of its original value. <u>This means a proper fraction will increase its value, while an improper fraction will decrease</u>. This rule is very important to denominate which fraction is bigger, as you can add numbers to compare. 3/2 > 4/3 > 13/12 > 1,013/1,012**

Quantitative Review
FDPs – Decimals and Fractions

- **Fractions**
  - **To divide fractions, use the reciprocal and then multiply. If you see double decker fractions (1/2/3/4), this is the same as 1/2 * 4/3 = 2/3**
  - **To compare fractions, cross-multiply and then compare the numerators. The numerator's fraction is bigger for the bigger fraction.**
  - **Example: 7/9 or 4/5? 7.5 = 35, 9/4 = 36. 36 is bigger, so the fraction 4/5 is bigger.**
  - **Take care when simplifying fractions. The denominator may never be splitted.**
  - **Example: (5x – 2y) / (x – y) cannot be simplified further**
  - **(6x – 15y) / 10 = 6x/10 – 15y/10 = 3x/5 – 3y/2**
- **Percents**
  - **Take care with the difference between "what is 30% of 80?", "75% of what number is 21?" and "90 is what percent of 40?"**
  - 30% of 80? **= x/80 = 30/100, x = 24**
  - 75% of what number is 21? **21/x = 75/100 = 3x = 84, x = 28**
  - 90 is what percent of 40? **90/40 = x/100, x = 225**

Quantitative Review
FDPs – Percents

- **Percent increase and decrease**
  - If a quantity is increased by x percent, then the new quantity is (100 + x)% of the original. Thus, to find a 15% increase, multiply by 1.15
  - If a quantity is decreased by x percent, then the new quantity is (100 – x)% of the original. To find a 15% decrease, multiply by 0.85
  - **Original +/- change = NEW**
  - **Data Sufficiency:**
  - **By what percent did the price of the book increase?**
  - **(1) The ratio original to new is 4:5**
  - **(2) the ratio change to new is 1:5**
  - **Both are sufficient, as this is the only information needed.**
- **Successive percents**
  - **Those are very common on GMAT. A price which is increased by x% and then decreased by y%, for example, is the same as <u>ORIGINAL . (1 + x%) . (1 – y%)</u>**
- **Chemical mixtures**
  - **A 500 mL solution is 20% alcohol. If 100mL of water is added, what is the new concentration of alcohol?**
  - **The original ratio 1:5 will become 1:6, so the concentration is 1/6 or 16,67%**

Quantitative Review
FDPs – Percents

- **FDPs – Mixed**
  - You will be often required to work with decimals, percents and fractions by the same time**. You have to be able to switch between them and understand the main advantages.** **<u>Fractions are good for calculations as we can cancel factors. Decimals are good for comparing sizes and for addition/subtraction.</u>**
- **FDPS and word translations**
  - **You must remember the most common:**
  - **X percent = X/100**
  - **Y is X percent of Z = Y = XZ/100 or Y/Z = X/100**
  - **G is 30% less than F → G = 0.7F**
- **FDPs – Advanced**
  - **Terminating decimals only have 2's and 5's as prime factors in their denominators.**
  - **If the denominator has a power of 10 minus 1 (9, 99), you will have repeating decimals**
- **Unknown digit problems**
  - **Those problems are tough, so use the following technique step by step:**

Quantitative Review
FDPs – Percents

- **Unknown digit problems**
  - **(1) Look at the answer choices first, to limit your search**
  - **(2) use other given constraints to rule out additional possibilities**
  - **(3) Focus on the units digit in the product or sum.**
  - **(4) Test the remaining choices**
  - **Example: AB x CA = DEBC. In this multiplication, each letter stands for a different non-zero digit, and A.B < 10; What is AB?**
  - **(A) 23 (B) 24 (C) 25 (D) 32 (E) 42**
  - **We can rule out choice C, as 2.5 = 10. Testing the others will result on E being the correct alternative.**
- **Fractions and Exponents and Roots**
  - **Proper fractions that are positive (0<x<1) will always yield an smaller number when raised to a power. Negative improper fractions when raised to an odd exponent will also yield a smaller number. All other fractions when raised to a power will yield a bigger number!**

Quantitative Review
FDPs – Percents

- **Percents and weighted averages**
  - Cereal K is 10% sugar and Cereal B is 2% sugar. What should be the ratio of them to produce a 4% sugar cereal?
  - **Technique: Pick a smart number for one of the quantities and call the other quantity x. For example, picking 100 grams for Cereal K:**
  - **$100*0.1 + 0.02*x = 0.04(100 + x) \rightarrow 10 + 0.02x = 4 + 0.04x$**
  - **$0.02x = 6 \rightarrow x = 300$**
  - **So, the ratio is 3 parts of Cereal B to each part of Cereal K, or 1:3**
- **Percent Change and Weighted Averages**
  - The revenue from pen sales was up 5%, but the revenue from pencil sales declined 13%. If the overall revenue was down 1%, what was the ratio of pen and pencil revenues?
  - $105 + 0.87x = 0.99(100 + x) \rightarrow 6 = 0.12x \rightarrow x = 50$
  - So, the ratio is 2:1
  - **Advanced question: A store sold 300 hammers and 12,000 nails on year 0 and 375 hammers and 18,000 nails on year 1. By approximately what percent did the ratio of hammers sold to nails sold decrease?**
  - **The ratio was 1/40 and now is 1/48. This may look like a**

Quantitative Review
FDPs – Percents

- **Percent Change and Weighted Averages**
  - **The ratio was 1/40 and now is 1/48. This may look like a 20% decrease but it is not! Take care!!**
  - **1/40 * 48 = 48/40 = 6/5**
  - **1/48 * 48 = 5/5**
  - **Therefore, the decrease was from 6 to 5, or 1/6 (16.7%)**
  - **Alternative method:**
  - **375/18,000/300/12,000 = 375/18,000 * 12,000/300 → 375/18 . 12/300**
  - **750/900 = 5/6. So, if NEW/ORIGINAL = 5/6, the decrease was 1/6.**
- **Estimating decimal equivalents**
  - **Example: Estimate 9/52**
  - **(1) 9/52 ≠ 9/50 = 18/100 = 0.18 (high estimate)**
  - **(2) 9/52 ≠ 9/54 = 1/6 = 0.167 (low estimate)**
  - **Now, we know that 9/52 is very close to the middle point of 0.167 and 0.18, which is approximately 0.173**